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INFLIGHT ESTIMATION OF GYRO NOISE*

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ABSTRACT

A method is described and demonstrated for estimating single-axis gyro noise levels in terms of the Farrenkopf model parameters. This is accomplished for the Cosmic Background Explorer (COBE) by comparing gyro-propagated attitudes with less accurate single-frame solutions and fitting the squared differences to a third-order polynomial in time. Initial results are consistent with the gyro specifications, and these results are used to determine limits on the duration of batches used to determine attitude. Sources of error are discussed, and guidelines for a more elegant implementation, as part of a batch estimator or filter, are included for future work.

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1. INTRODUCTION

1.1 WHY WORRY ABOUT GYRO NOISE?

The usual batch approach to attitude estimation ignores the contribution of propagation to solution error. As long as the gyro angular rates are perfect, such simplification is justified. Of course, true perfection is never achieved. Even if systematic errors due to biases, scale factors, and misalignments are removed, there is still a random component of the error, which is called "noise." Some noise is completely random. Its value at one instant says nothing about that at the next. This is "white noise," and it shows up as a jittery line (Figure 1). Closer inspection reveals slowly varying biases on the gyro rates. Although both of these random errors average to zero, at least over infinite time, their instantaneous effects are not zero and tend to increase in size with propagation time.

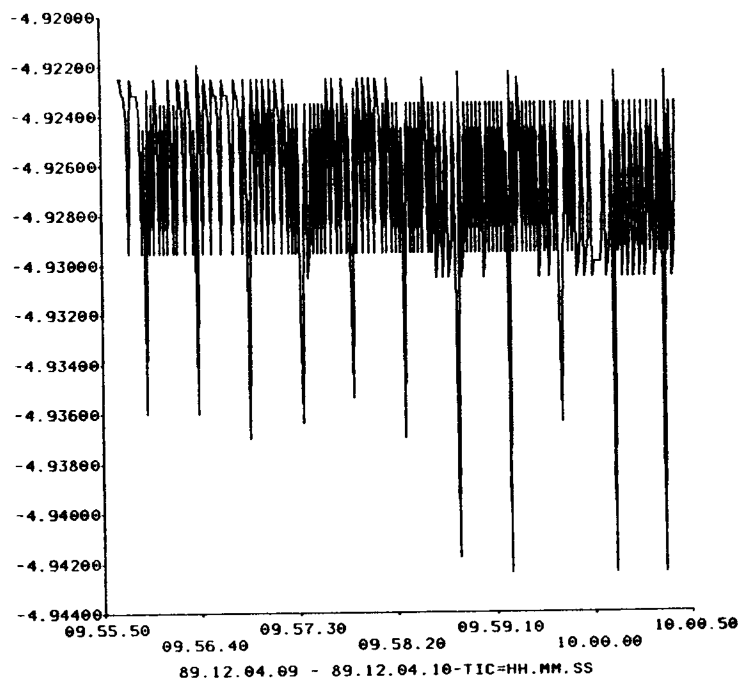


Figure 1. Noisy COBE X-Gyro Rates

To control the effect of gyro noise, the propagation time and batch length must be limited. Batch length depends on the frequency and accuracy of the observations and the gyro noise levels. Kalman filtering addresses the propagation error problem more elegantly by gradually forgetting past observations, and it can incorporate models that automatically account for random process noise. Whether a batch estimator or a filter is used, to obtain the best results the gyro noise level must be known.

When the impact of gyro noise was studied prior to launch of the Cosmic Background Explorer (COBE), the X-gyro scale factor instability appeared to limit batch duration to

20 minutes, or one-fifth of an orbit (Reference 1). This was a serious constraint, because the batch estimator is often used to average out sensor errors that vary at the orbit rate. Short batches would not be as effective for this purpose. To quantify the impact of gyro noise systematically, the COBE ground support software was enhanced to estimate noise parameters, attitude, and calibrations.

1.2 THE APPROACH

Gyro noise shows up in several ways. It is directly observable in the rate data, as Figure 1 illustrated. The most obvious noise measure is the variance of the rate, but variance includes real motion and so overestimates the noise. Another method is to set some cutoff frequency above which the spacecraft cannot move and then find the variance of the rate minus the components below that frequency. However, this leaves out any random drift of the rate and so underestimates the noise. A third method is to obtain limits on the total rate noise. The long-term variation of the rate is also observable in the history of the drift rate bias. Computing the bias for many orbits over several days, as in Figure 2, provides information from which to estimate the very low frequency part of the rate noise.

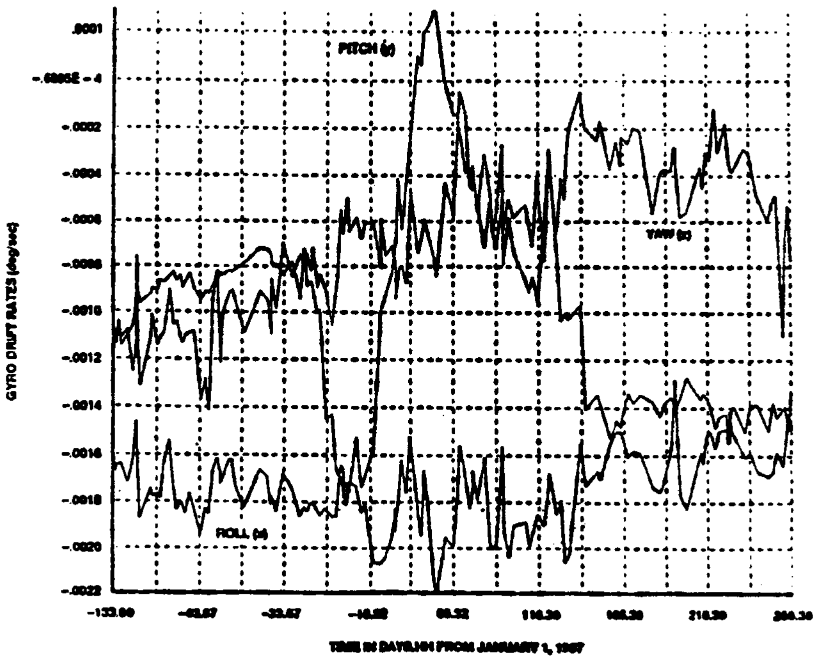


Figure 2. History of the ERBS Gyro Biases

Unfortunately, estimating noise directly from the rates does not give a very complete picture of the gyro noise. This is particularly true over the frequency midrange from 100 hertz (Hz) to 1 revolution per orbit, where it is difficult to distinguish between motion and noise. To distinguish motion from noise, one has to take advantage of the sensor observations, and this means working with attitudes instead of rates.

COBE has two attitude estimators; the batch estimator uses the gyros, while the single-frame estimator does not. By comparing the two estimates, one can isolate the effects of gyro noise. The squared difference between them becomes an observation of the combined constant variance in each and the growing variance due to propagation error. With a model for how that variance grows with time, one can construct an estimator for the model parameters.

2. THE GYRO NOISE MODEL

2.1 OVERVIEW

One popular method of estimating the effect of gyro noise is the Farrenkopf model (Reference 2). For the COBE rate-integrating gyros operating in rate mode, the output is integrated only over the 1-second interval between gyro samples. These incremental angles, through which the spacecraft rotates between samples, are just like samples of the angular rate times the constant sampling interval. For this reason, the gyros are treated here as rate gyros rather than rate-integrating gyros. The model includes four noise sources, and for each noise source it computes a transfer function relating the noise to the output angle. From the inverse transfer functions, an expression is obtained to relate the variance of the noise sources to that of the total rotation angle obtained from the sum of the incremental angles.

2.2 THE PHYSICS OF RATE GYROS—OPEN LOOP

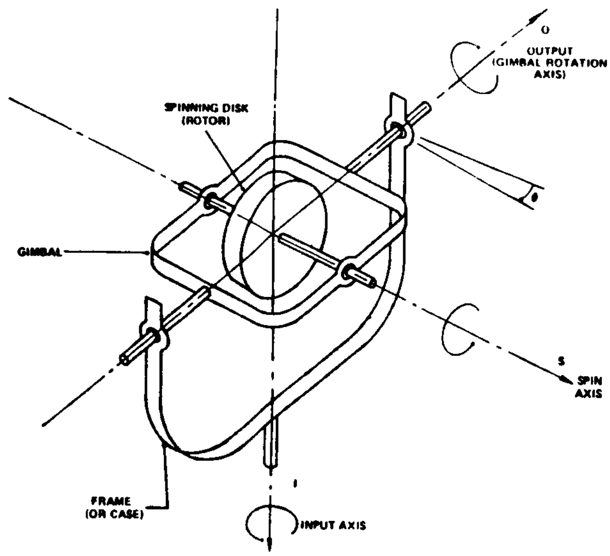
Rate gyros do not have their angular momentum vectors fixed in inertial space but are forced to rotate with the body, as shown in Figure 3. A torque must be applied to change the pointing direction of the rotor, and this torque tilts the rotor up or down in proportion to the input rotation angle.

Euler's equation for rigid body motion describes the behavior of the open loop non-rebalanced gyro. Here, M is used for the torque about the output axis, and H is used for the angular momentum of the rotor, which is assumed to have constant magnitude. The dot denotes differentiation with respect to time.

$$M = \dot{H} \tag{2-1}$$

The applied torque M consists of two parts. Viscous damping from the gimbal bearing is equal to the product of the damping coefficient C with the output angular rate $\dot{\theta}$. In addition, torque is applied by the gimbal assembly to change the rotor direction. For small θ, this torque is equal to the product of the input angular rate ω and the rotor momentum H.

$$\omega H \cos(\theta) \approx \omega H \tag{2-2}$$



Source: L. Fallon, III, "Gyroscopes" (Section 6.4), *Spacecraft Attitude Determination and Control*, J. R. Wertz, ed. Dordrecht, Holland: D. Reidel Publishing Company, 1980.

Figure 3. Single Axis Gyro Without Rebalance

The time derivative of the angular momentum is the transverse moment of inertia for the rotor I times the angular acceleration of output angle $\ddot{\theta}$.

$$-C\dot{\theta} + \omega H = I\ddot{\theta} \tag{2-3}$$

Because the system is linear, it is convenient to use Laplace transform notation where "s" indicates differentiation and "1/s" indicates integration. Written in Laplace transform notation, the open loop equation is

$$-sC\theta(s) + H\omega(s) = s^2 I\theta(s) \tag{2-4}$$

This can also be represented in a block diagram, as in Figure 4.

The transfer function of $\theta(s)$ with respect to $\omega(s)$ is

$$\frac{\theta(s)}{\omega(s)} = \frac{H}{sC\,(1 + s\tau_o)} \tag{2-5}$$

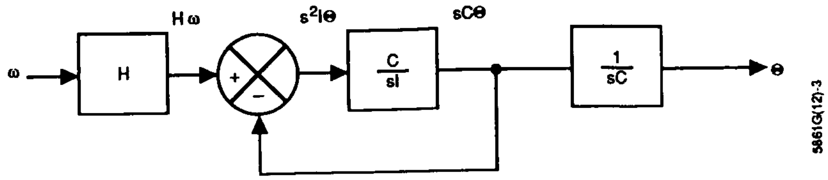


Figure 4. Open Loop Gyro Model

where τ_o is the open loop gyro time constant and, for COBE, is equal to 0.7 x 10^{-3} seconds (Reference 3).

$$\tau_o = I/C = 0.7 \times 10^{-3} \text{ seconds} \tag{2-6}$$

The impulse response or inverse transform for this open loop transfer function is

$$\theta(t) = (1 - \tau_o e^{-t/\tau_o}) \frac{H}{C} \tag{2-7}$$

Because the integration time of 1 second is much longer than τ_o, the second term can be neglected. This corresponds to dropping the inertial term from the original differential equation and the transfer function to give

$$\omega H \approx C\dot{\theta} \tag{2-8}$$

and

$$\frac{\theta(s)}{\omega(s)} \approx \frac{H}{sC} \tag{2-9}$$

The tilt angle θ, however, is not the rotation about the input axis that is sought. To this end, if ω is recognized as the time derivative of the input rotation angle ϕ

$$\omega = \dot{\phi} \tag{2-10}$$

ϕ can be obtained by integrating the differential equation.

$$\phi = \int_0^T \omega \, dt \approx \int_0^T C\dot{\theta}/H \, dt \tag{2-11}$$

If the input rate ω is assumed constant over the 1-second sampling interval, the integral can be replaced by a simple proportional relationship between the input and output angles.

$$\phi \approx C\dot{\theta}/H \tag{2-12}$$

Because this constant of proportionality changes rapidly when the tilt angle is large, most rate gyros have a feedback control loop to rebalance the rotor and keep the tilt small.

2.3 TORQUE REBALANCING—CLOSED LOOP

In torque-rebalanced rate gyros, such as those on COBE, the response of the gyro to an input rotation is damped by a viscous torque, and a restoring torque is applied to return the gyro to the null position in the spacecraft frame. The control loop for rebalancing the gyros has as input the incremental output angle $\Delta\theta$, or the difference between the current tilt of the rotor and the tilt at the previous time step.

This incremental tilt angle $\Delta\theta$ is divided by the time between samples, multiplied by the rotor angular momentum, and is fed back as a torque. The output is actually a pulse-width modulated (PWM) signal in which the pulse width is proportional to the tilt of the gyro from the null position. To calculate the total input angle ϕ through which the spacecraft has moved, the pulses are summed and scaled appropriately.

Adding feedback proportional to the output angle $\Delta\theta$ and making the simplifications discussed above gives the Farrenkopf model shown in Figure 5.

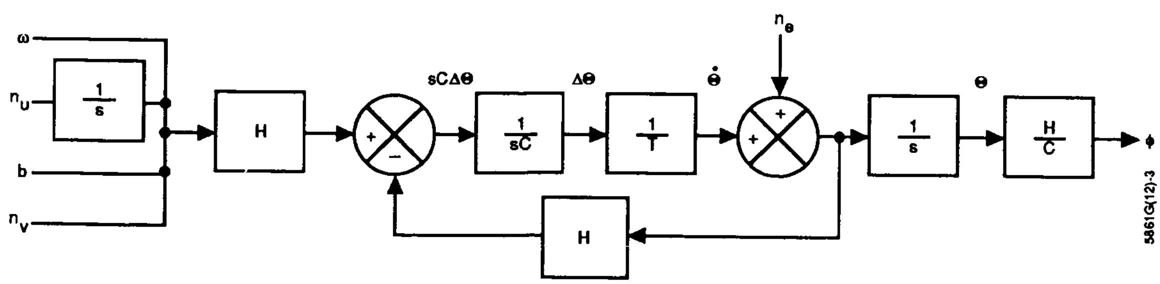


Figure 5. Closed Loop Rate Gyro Model

Ignoring the error sources for the moment, the system equation is

$$H\omega(s) - \frac{H}{T}\Delta\theta(s) = sC\Delta\theta(s) \tag{2-13}$$

As shown in Figure 5, $H\Delta\theta(s)/T$ is subtracted from $H\omega(s)$ leaving $sC\theta(s)$. This quantity is divided by sC to give the incremental angle $\Delta\theta(s)$. Because the feedback signal must be a torque, $\Delta\theta(s)$ is divided by the sampling interval T to produce an average rate $\theta(s)$ and is then multiplied by the rotor angular momentum H.

$$\dot{\theta} \equiv \Delta\theta/T \tag{2-14}$$

The resulting transfer function relating the input rate $\omega(s)$ to the output rate $\dot{\theta}(s)$ is

$$\frac{\dot{\theta}(s)}{\omega(s)} = \frac{1}{1 + sCT/H} = \frac{1}{1 + s\tau_c} \tag{2-15}$$

The impulse response for this system is

$$\dot{\theta}(t) = e^{-t/\tau_c} \tag{2-16}$$

where the closed loop time constant τ_c is

$$\tau_c \equiv CT/H \tag{2-17}$$

For COBE, the closed-loop time constant is 0.03 second, which yields a bandwidth of 5 Hz for the rebalanced system. For timescales much longer than the closed-loop time constant τ_c, the rebalanced rate gyro passes the input rate straight through. This can be represented by eliminating the loop from Figure 5.

2.4 ERROR SOURCES AND EXPECTATIONS

The gyro model includes the following four error sources that enter the control loop as either input or output errors (Reference 4):

- Rate bias (b), which is a constant that is imprecisely known
- Float torque noise (n_v), which is white noise on the rate bias
- Float torque derivative noise (n_u), which is the "strength" of the rate bias random walk
- Electronic noise (n_e), which is white noise on the incremental angle θ

Although the rate bias is constant, it is imprecisely known and when used for propagation introduces error. It can be estimated, but it can never be known perfectly. Because it is constant, the bias autocorrelation, which is needed for predicting the variance of the integrated angle ϕ, is also a constant. The variance is obtained by setting the two times λ and τ equal in the expression for the autocorrelation. E[...] is used here as the expectation operator.

$$E[b(\lambda)b(\tau)] = \sigma_b^2 \tag{2-18}$$

The float torque, float torque derivative, and electronic noise sources are zero mean and white, meaning that their value at one time is independent of their value at any other time. This is an idealization and is not true if the sampling is done at a rate above the highest frequency component of the white noise. Although the idealization is used here, it is understood that this applies to the COBE gyro sampling rate of 1 Hz.

The white noise approximation is indicated by the Dirac delta function $\delta(t)$, which is zero for all t except zero, and which integrates to 1 over an interval including zero. In addition, these three white noise sources are completely uncorrelated among themselves. This is reflected in the equation below using the Kronecker delta function δ_{ij}, which is zero when i is not equal to j and 1 when it is.

$$E[n_i(\tau)] = 0 \quad (i = v, u, e) \tag{2-19}$$

$$E[n_i(\lambda)\, n_j(\tau)] = \sigma_i^2\, \delta_{ij}\, \delta(\lambda - \tau) \tag{2-20}$$

The names float torque and float torque derivative noise are holdovers from the days when the rotors were floated in a liquid for rebalance. Although the physical interpretation of these sources is different for the COBE dry-tuned gyros, the effect is the same and the traditonal names are preserved.

The transfer function for the summed output angle θ with respect to n_v and b is 1/s times the transfer function for the output angular rate θ derived above. This corresponds to the final integration of the sampled rates to give the total rotation.

$$\frac{\theta(s)}{\omega(s)} = \frac{\theta(s)}{N_v(s)} = \frac{\theta(s)}{B(s)} = \frac{\dot{\theta}(s)}{s\omega(s)} = \frac{1}{s(1 + s\tau_c)} \tag{2-21}$$

The corresponding impulse response is

$$g_v(t) = g_b(t) = 1 - e^{t/\tau_c} \tag{2-22}$$

The transfer function for n_u is 1/s times that for n_v.

$$\frac{\theta(s)}{N_u(s)} = \frac{\theta(s)}{sN_v(s)} = \frac{1}{s^2(1 + \tau_c s)} \tag{2-23}$$

which has the impulse response

$$g_u(t) = \tau_c e^{-t/\tau_c} + t - \tau_c \tag{2-24}$$

The output transfer function for n_e is

$$\frac{\theta(s)}{N_e(s)} = \frac{\tau_c}{1 + \tau_c s} \tag{2-25}$$

with impulse response

$$g_e(t) = e^{-t/\tau_c} \tag{2-26}$$

The response of the system to each of these inputs is the convolution integral of the input with the appropriate impulse response function. Because the system is linear, the combined response is the sum of the individual responses. This and the fact that the noise sources are independent imply that the variance of the total rotation angle equals the sum of the individual variances.

$$\mathrm{var}(\theta) = \mathrm{var}(\theta_e) + \mathrm{var}(\theta_v) + \mathrm{var}(\theta_b) + \mathrm{var}(\theta_u) \tag{2-27}$$

Since each of the noise sources has zero mean, the expected response is also zero mean. This is because the expectation operator can be brought inside the convolution integral to act upon n_i, as in the example below. The expected value of n_i is zero, and this makes the expected response zero as well.

$$E[\theta_i] = E\left[\int_0^t n_i(\tau)\, g_i(t-\tau)\, d\tau\right] = 0 \tag{2-28}$$

The expected autocorrelation is the expectation of the product of two convolution integrals. Because the expected value is zero, there is no square of the average to subtract. As before, the variance is obtained by evaluating the autocorrelation at a single time t.

$$\mathrm{var}(\theta_i) = E\left[\left(\int_0^t n_i(\lambda)\, g_i(t-\lambda)\, d\lambda\right)\left(\int_0^t n_i(\tau)\, g_i(t-\tau)\, d\tau\right)\right] \tag{2-29}$$

These integrals can be combined and the expectation operator brought inside to give

$$\mathrm{var}\,(\theta_i) = \int_0^t g_i(t-\lambda) \int_0^t E[n_i(\lambda)\, n_i(\tau)]\, g_i(t-\tau)\, d\tau d\lambda \tag{2-30}$$

Formal integration of these expressions using the appropriate autocorrelations for the four error sources yields the following expressions for their contributions.

$$\mathrm{var}(\theta_b) = \sigma_b^2[t - \tau_c\,(1 - e^{-t/\tau_c})]^2 \tag{2-31}$$

$$\mathrm{var}(\theta_v) = \sigma_v^2(t - 3\tau_c/2 + 2e^{-t/\tau_c} - \tau_c e^{-2t/\tau_c}/2) \tag{2-32}$$

$$\mathrm{var}(\theta_u) = \sigma_u^2(t^3/3 - t^2\tau_c + t\tau_c^2 + \tau_c^3 - 2t\tau_c^2\, e^{-t/\tau_c} - t_c^3\, e^{-2t/\tau_c}) \tag{2-33}$$

$$\mathrm{var}(\theta_e) = \sigma_e^2\, \tau_c(1 - e^{-2t/\tau_c}) \tag{2-34}$$

Given that the closed-loop time constant τ_c is 0.03 second and a typical propagation time-span is thousands of seconds long, the total variance is well approximated by a cubic polynomial in time.

$$\mathrm{var}(\theta) \approx \tau_c \, \sigma_e^2/2 + \sigma_v^2 t + \sigma_b^2 t^2 + \sigma_u^2 t^3/3 \qquad (2\text{-}35)$$

Finally, this variance of the summed gyro output angle should be multiplied by (C/H)2 to convert the variance of θ to the variance of ϕ, the angle through which the spacecraft has rotated. For the COBE gyros, this factor is 1 and so the variances of θ and ϕ are equal.

$$\mathrm{var}(\theta) = \mathrm{var}(\phi) \qquad (2\text{-}36)$$

3. ESTIMATES OF THE GYRO NOISE

3.1 IMPLEMENTATION AND PRACTICE

Because gyro noise estimation was not planned for from the beginning, it is done with the existing COBE software, as shown in Figure 6. As a result, instead of using sensor observations directly, the fine attitude determination subsystem (FADS) gyro-propagated attitude is used as the observation and the coarse attitude determination subsystem (CADS) nongyro single-frame solution is used as a reference. The two attitudes are converted to their Euler angle representations in the quality assurance subsystem (QA), and the squared difference in the yaw angle serves as a sample of the propagation error variance at that time.

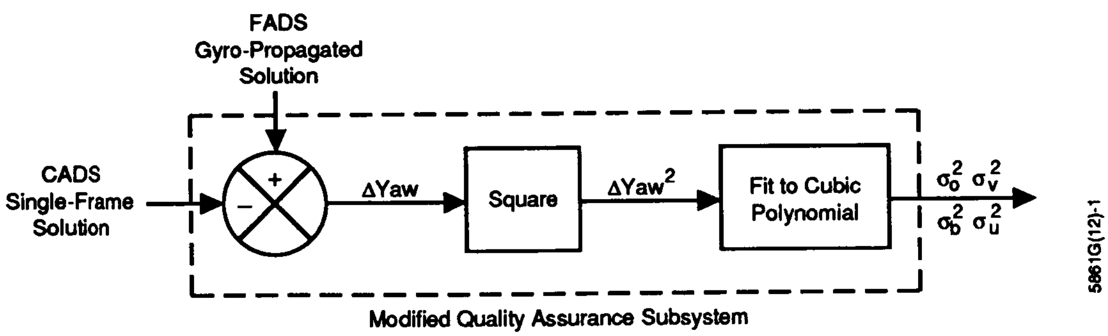


Figure 6. Spin Gyro Noise Estimator

The variation of this sampled variance is fit to a cubic polynomial function of time, in keeping with the model developed in the last section. The one difference is that due to the large single-frame solution noise, the constant term $\tau_c\sigma_e^2$ is more a reflection of this

single-frame error than of electronic signal generator noise as in the Farrenkopf paper (Reference 2). For this reason, the constant variance is referred to hereafter as σ_0^2.

One problem with this approach is that, due to the 4-degree roll of the spacecraft, the spin axis gyro also picks up some of the pitch motion. If gyro noise estimation had been planned for before the ground support software was written, it could have been implemented more cleanly.

Gyro noise is usually measured by putting the gyro on a turntable and comparing the integrated displacement angle with the measured angle (Reference 5). For accuracy, the test can be conducted over many days. Although not impossible in flight, such long tests are awkward since they require large data sets and long processing times. Instead, shorter timespans are employed and the results averaged to improve accuracy.

Averaging helps more in finding σ_v than it does in finding σ_u. This is because the variance from σ_v grows linearly with time, while that from σ_u grows as the third power of time. One-tenth the propagation time provides one-tenth the variance and therefore one-tenth the information about σ_v. Repeating the short test 10 times provides the same information as one long test. For σ_u, however, one-tenth the time provides one-thousandth the information. Repeating the test over 10 intervals provides only a small portion of the information about σ_u that the longer run provides.

As described above, gyro noise is assumed to cause growing errors in the propagation, which are seen by comparing the gyro-propagated solutions with the single-frame solutions. The single-frame solutions are themselves noisy and susceptible to large localized errors, as shown in Figure 7. Thus, long timespans covering several orbits are necessary. As long as the large errors are constant from orbit to orbit, the gradual increase in error due to gyro noise is still discernable.

Another problem arises because of the least squares optimization criterion used in producing the gyro-propagated attitude. Because two medium-size errors at the ends of a batch are preferable to no error at one end and a large error at the other, the minimum propagation error is apt to be at the center rather than the start of the batch. This is also visible in Figure 7.

Instead of fitting one monotonically increasing polynomial to the entire timespan, the batches are broken into halves. In finding coefficients for the first half of the data, time is thought of as going backward from the middle to the start of the batch. This corrects the signs of the odd power coefficients, which would otherwise have the wrong sign. An alternative to this would be to propagate from a single-frame solution. In this case, the constant term σ_0^2 would be set to zero.

3.2 RESULTS AND COMPARISON WITH SPECIFICATIONS

Few results have been collected as of this time. The following coefficients come from the example of Figure 7. They represent the estimated noise from a 5-hour, 38-minute timespan.

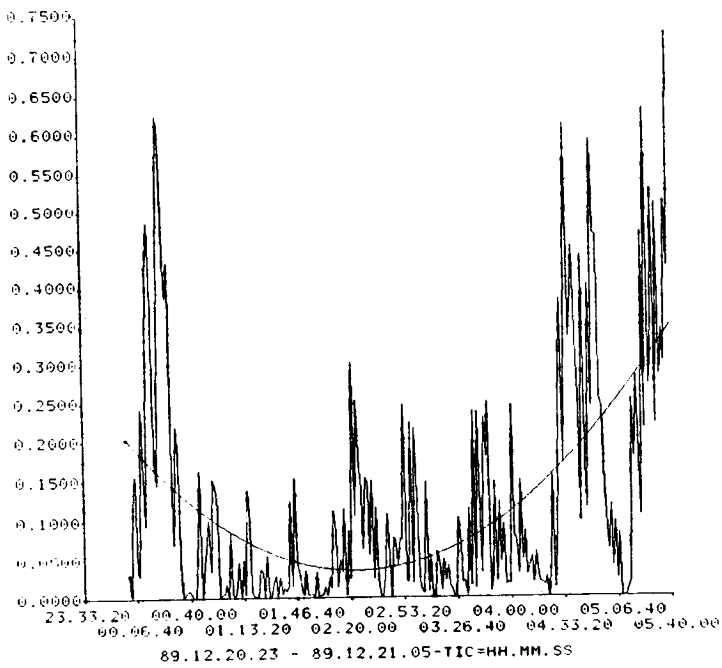


Figure 7. Typical Yaw Differences and Their Cubic Fit

Single Frame Noise	σ_0^2	0.1464 deg^2
Float Torque Noise	σ_v^2	0.2196×10^{-4} deg^2/s
Gyro Bias	σ_b^2	0.4006×10^{-8} deg^2/s^2
Float Torque Derivative Noise	σ_u^2	-0.4458×10^{-12} deg^2/s^3

The first number represents the combined variance of the batch and single-frame yaw solutions, and appears to be dominated by the large spikes in the yaw differences. The second coefficient represents the white noise on the gyro rate. For samples taken every second, this implies an error of 0.005 degree per second$^{1/2}$. The 24 arc-second telemetry quantization explains 0.002 degree per second$^{1/2}$ of that number, and the rest comes from other sources. The third coefficient is the variance on the assumed gyro bias and is 0.2 degree per hour, which is about the accuracy of the typical bias solution. Finally, the fourth coefficient is negative, indicating that it is not well estimated over this timespan. A longer batch may be necessary to observe it.

Expected worst case values of σ_v^2 and σ_u^2 can be backed out of the following COBE specifications (Reference 1).

- Root mean square gyro drift rate noise error will be less than 0.002 degree per second
- Gyro rates will be sampled at 1-second intervals (science format at 4 kilobits per second)

- X-gyro telemetry quantization will be 24 arc-seconds per count, sampled once per second
- X-gyro scale factor will be stable to 50 parts per million over 30 minutes
- Nominal spin rate will be 4.8 degrees per second

If the drift error is assumed to be the result of integrating the float torque noise over the 1-second sampling interval, it implies that the inherent float torque noise variance is 0.4×10^{-5} deg^2/s. The telemetry quantization contributes the same amount to the effective float torque noise, thus doubling its strength to 0.8×10^{-5} deg^2/s.

To estimate the float torque derivative noise, the scale factor stability is multiplied by the spin rate to give the spin rate stability. Assuming this error to be the standard deviation of the rate at the end of the specified 30-minute period, the float torque derivative noise would have to be 0.3×10^{-11} deg^2/s^3.

Float Torque Noise	σ_v^2	0.8×10^{-5} deg^2/s
Float Torque Derivative Noise	σ_u^2	0.3×10^{-11} deg^2/s^3

The estimated value of σ_v^2 is 2-1/2 times that specified, but more sample propagations are needed before the gyro performance is questioned. Besides, there may be other sources of error outside the gyro that may behave like the float torque noise. The estimate of σ_u^2, while not valid because of the sign, is the right order of magnitude. It is likely that the scale factor stability and other float torque derivative noise sources are well within specifications and are too small to be observed over a 5-hour, 38-minute propagation.

3.3 USING THE RESULTS

For a Kalman filter that estimates the attitude and rate bias, the noise standard deviations can be used to compute the state noise covariance matrix (Q), which is added to the state covariance matrix (P) after each propagation step (T) (Reference 1). This controls how quickly the old state estimate is forgotten.

$$Q = \begin{bmatrix} \sigma_v^2 T + \sigma_u^2 T^3/3 & -\sigma_u^2 T^2/2 \\ -\sigma_u^2 T^2/2 & \sigma_u^2 T \end{bmatrix} \quad (3\text{-}1)$$

For the batch estimator, the only way to control propagation error is to limit the batch length τ_B. The estimator can be modeled as an averaging operation and incorporated into a simplified random process model as shown in Figure 8.

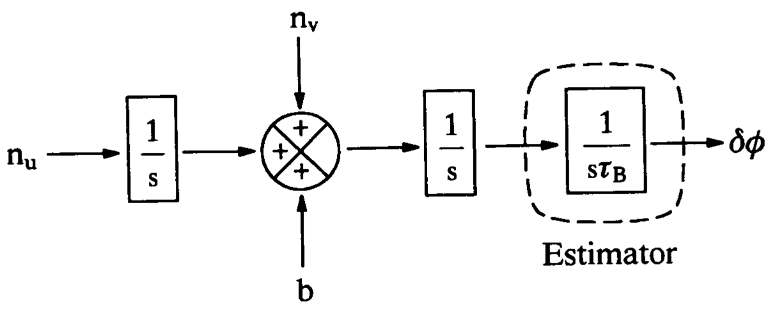


Figure 8. Propagation Error and Batch Estimation

If the observations have variance σ_0^2 and come at a rate of k per second, the epoch error $\delta\phi$ has the following variance:

$$\text{var}\,(\delta\phi) = \frac{\sigma_0^2}{k\,\tau_B} + \frac{\sigma_v^2\,\tau_B}{3} + \frac{\sigma_b^2\,\tau_B^2}{4} + \frac{\sigma_u^2\,\tau_B^3}{20} \tag{3-2}$$

For the COBE float torque noise estimate and azimuth observations with variance 0.02 deg^2 made every 0.5 sec, the epoch solution variance is minimal when the batch is 55 seconds long. This criterion ignores the inconvenience of doing many short batch solutions. A more realistic way to use this estimate is to choose batch length to keep the epoch solution variance within acceptable limits. For the COBE definitive accuracy requirement of 3 arc-minutes (3σ), this restricts batches to 70 sec. For routine attitude determination where the requirement is 1 deg (3σ), batches of up to 50 minutes are acceptable.

4. HOW NOISE ESTIMATION COULD BE DONE BETTER

4.1 THE DIFFICULTY OF OBSERVING σ_u^2

To observe σ_u, one must propagate for a long time. The current batch estimator, which stores the data for the entire batch in memory, makes this very expensive. Skipping points permits longer batches but may introduce additional error that is not really in the gyro data. A better way would be to propagate open loop and estimate the noise parameters as one goes along. In this way, one could process long timespans without as much overhead.

4.2 A FILTER IMPLEMENTATION

One solution is to incorporate the noise estimation in a Kalman filter, as shown in Figure 9. This would facilitate long propagations, because the end state could be used as the

starting point for the next timespan. This could also be done with a modified COBE batch estimator if it were operated so as to produce a continuous attitude history. In either case, the partial derivatives of the observations with respect to the noise parameters would also have to be passed in with the attitude.

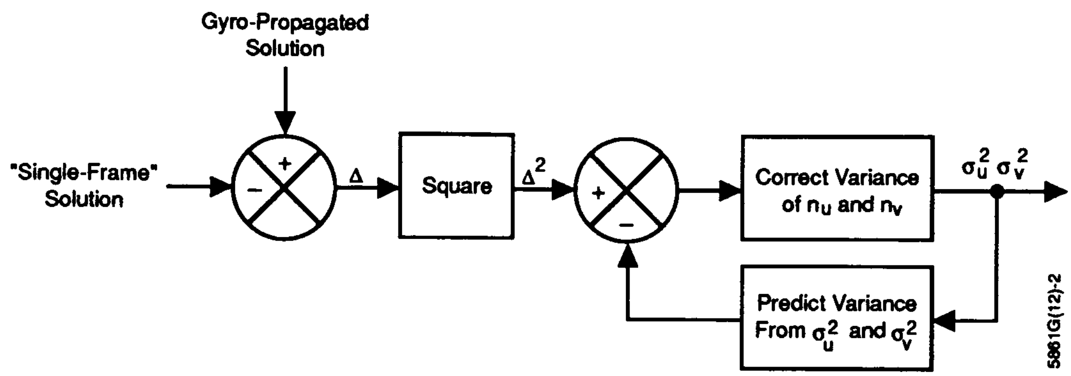


Figure 9. Filter Noise Estimator

The observations of the gyro noise source variances would come from the propagated attitude (a_c). The reference would be the attitude estimated from sensor observations (a). The noise model would be the same as that used earlier.

$$[a - a_c]_i^2 = \sigma_0^2 + \sigma_v^2 t_i + \sigma_b^2 t_i^2 + \sigma_u^2 t_i^3/3 \tag{4-1}$$

Ideally, the estimated attitude would be computed independently of the gyro rates. This is no problem for COBE, which has continuous attitude observability. For other spacecraft, one might have to settle for fewer observations of the propagation error due to the lack of a complete reference attitude. Alternatively, one might accept use of the gyros to produce the reference attitude as long as the state noise covariance (Q) was large enough that old observations were forgotten quickly.

The approach could also be extended to all three axes if a model could be developed for the contribution of the noise on each gyro axis to each propagated angle. Those contributions would depend on the actual angular velocity and so would have to be computed numerically over the duration of the propagation, just as the epoch-to-current-attitude propagation matrix and gyro bias variational matrix are computed now in the batch estimator.

5. CONCLUSIONS

The method presented here for estimating gyro noise levels has been derived from a simple but theoretically based gyro model and has been shown to provide reasonable

results. Although these results are quite believable, this single sample should not be given too much weight. Many such runs, or even longer ones, should be made and the results averaged before being accepted. After all, ground tests of the gyros last for days, and at least this much time is needed in flight, where the reference attitude is so much less accurate. To this end, the filter version of the noise estimator should be considered.

The next step in estimating noise should be to look at the other manifestations of noise, such as the drifting of gyro biases over the course of days and weeks and the spectral decomposition of the gyro rates themselves. The former is a comparatively long timescale variation, which should be compatible with the shorter effects examined here. The latter is a shorter timescale approach, which should also agree with the present estimates of the gyro noise.

Perhaps even more important than continuing to study the problem is putting these noise estimates to use in tuning Kalman filters and setting batch lengths for more enlightened attitude determination. If these results prove helpful in this regard, a new system that estimates gyro noise along with attitude could be built that would provide more information about gyro performance more conveniently than is now possible.

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